## Estimate surface area of a room

Program Task: Estimate the surface area of the ceiling, walls and floor to determine how much insulation, drywall, paint, and flooring is needed to renovate a room.

## Program Associated Vocabulary:

DIMENSION, ESTIMATE, INCH, FOOT, YARD, SQUARE INCH/FOOT/YARD, COVERAGE

## Program Formulas and Procedures:

When carpenters are involved in a building project, they have to determine surface area to purchase the correct quantity of material to complete the project. The most common shape that carpenters are involved with is the rectangular prism. A rectangle prism compares to a room. It has a base (floor); it has four sides (four walls) and it has a top (ceiling). In math the top is also referred to as a base.
Many rectangular prisms make up a house. But in order to estimate needed material each room needs to be looked at individually.

## Example:

Below is a room, a rectangular prism; find the surface area using the following formula:

$$
S A=2 l w+2 w h+2 l h
$$


width $=14^{\prime}$
length $=28^{\prime}$
height $=9^{\prime}$
$\mathrm{SA}=2(28)(14)+2(14)(9)+2(28)(9)$
SA $=784+252+504$
$\mathrm{SA}=1540 \mathrm{ft}^{2}{ }^{2}$
Carpenters will use this information to determine how much surface area of flooring is needed:
$\mathrm{SA}=\mathrm{I} \times \mathrm{w} \quad \mathrm{SA}=28^{\prime} \times 14^{\prime}=392 \mathrm{ft}^{2}{ }^{2}$

Carpenters will use the information when purchasing paint if they were to paint the ceiling and the walls.
SA = 1540' - floor =
SA $=1540^{\prime}-392^{\prime}=1148^{\prime} \mathrm{sq} \mathrm{ft}$.

## Apply geometric concepts to model and solve real world

 problems
## PA Core Standard: CC.2.3.HS.A. 14

Description: Apply geometric concepts to model and solve real world problems.

## Math Associated Vocabulary: <br> AREA, CROSS SECTION, LENGTH, WIDTH, RECTANGLE, BASE, HEIGHT, RADIUS, RECTANGULAR PRISM

## Formulas and Procedures:

Surface Area:

Cylinder:
$\mathrm{SA}=2 \pi \mathrm{r}^{2}+2 \pi \mathrm{rh}$


Cone:
$\mathrm{SA}=\pi \mathrm{r}^{2}+\pi \mathrm{r} \sqrt{\left(\mathrm{r}^{2}+\mathrm{h}^{2}\right)}$


Rectangular Prism:
SA $=2 \mathrm{lw}+2 \mathrm{wh}+2 \mathrm{hl}$


Sphere:
$\mathrm{SA}=4 \pi \mathrm{r}^{2}$


Pyramid:
SA $=($ area of the base $)+1 / 2 \ell$ (perimeter of base)
$\mathrm{b}=$ base, $\mathrm{h}=$ height, $\ell=$ slant length


Example: Find the surface area of the cylinder below.

$\mathrm{r}=1 / 2 \times 38^{\prime \prime}=19^{\prime \prime} \mathrm{h}=60 \prime$
Cylinder $\mathrm{SA}=2 \pi \mathrm{r}^{2}+2 \pi \mathrm{rh}$

$$
\mathrm{SA}=2 \pi(19)^{2}+2 \pi(19)(60)
$$

$$
\mathrm{SA}=722 \pi+2,280 \pi
$$

$$
\mathrm{SA}=3,002 \pi
$$

$$
\mathrm{SA}=9431.06 \mathrm{in}^{2}{ }^{2}
$$

## Instructor's Script - Comparing and Contrasting

In most cases a carpenter must work with rectangular prisms, but in more complex architectural plans, other shapes are used for dramatic appeal.

## Common Mistakes Made By Students

Using incorrect formula: Students may use an incorrect formula to solve a problem. To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

When dealing with the surface area of cone or a pyramid the instructor needs to stress the difference between the height (h) and slant height $(l)$.

Using consistent units: If the problem asks for the answer in square feet instead of square inches, be sure to either convert your given measurements into feet first (inches $\div 12=$ feet) or convert your square inch answer into square feet (sq. inches $\div 144=$ sq. feet).

Not "removing" unnecessary surface areas from calculations: Depending on the problem, not all surface areas included in formula may be needed. Identify the areas that are required for the calculation and remove from formula as needed.

## CTE Instructor's Extended Discussion

Surface area is often used when estimating construction projects. The following are just some of the examples when carpenters use surface area for estimating: foundation concrete sealer, plywood wall sheathing, T \& G flooring, roof sheathing, drywall, insulation, paint, floor coverings, siding and roofing.

| Problems Career and Tec | Solutions |
| :---: | :---: |
| 1. A customer has asked you to paint a grain silo. The silo's radius $=15^{\prime}$ and the height $=50^{\prime}$. What is the total surface area of the top and sides of the silo that will be painted? |  |
| 2. You are asked to paint the ceiling and walls of a room. The room is 10 ' wide $\mathrm{x} 15^{\prime}$ long x $10^{\prime}$ high. Calculate the surface area so that you can determine the gallons of paint needed. A gallon of paint covers 175 sq. ft. |  |
| 3. You need to paint two decorative spheres with $\mathrm{d}=10^{\prime}$ at the entrance to the new mall. A gallon of paint covers 175 sq. ft. How many gallons of paint will you need? |  |
| Problems Related, Gene | Solutions |
| 4. You need fabric to cover a 4 -sided pyramid with base sides of $12^{\prime} \&$ slant length of $20^{\prime}$. How many square feet of fabric will you need to cover the pyramid? How many square yards? Note: $1 \mathrm{yd.}^{2}=9 \mathrm{ft.}^{2}$. |  |
| 5. One soup can has a radius $=3$ " and height $=4$ "; another soup can has a radius $=4$ " and height $=3 "$ ". Which can has a greater total surface area? |  |
| 6. A size 7 regulation basketball has a $\mathrm{d}=9.3^{\prime \prime}$ ". A size 6 regulation basketball has a $\mathrm{d}=9.07^{\prime \prime}$. What is the surface area of each basketball? |  |
| Problems PA Core | Solutions |
| 7. Find the surface area of a cylinder with a diameter of $13.75^{\prime}$ and a height of 28.45 '. |  |
| 8. Find the surface area of a sphere that has a diameter of 27.75". |  |
| 9. Find the total surface area of a cone with base diameter of $15.50^{\prime \prime}$ and a height of 22 ". |  |


| Problems Career and Tech | nical Math Concepts Solutions |
| :---: | :---: |
| 1. A customer has asked you to paint a grain silo. The silo's radius $=15^{\prime}$ and the height $=50^{\prime}$. What is the total surface area of the top and sides of the silo that will be painted? | Cylinder SA $=2 \pi \mathrm{r}^{2}+2 \pi \mathrm{rh}$ <br> But only the top is need, so: $\mathrm{SA}=\pi \mathrm{r}^{2}+2 \pi \mathrm{rh}$ $\begin{aligned} & \mathrm{SA}=\pi(15)^{2}+2 \pi(15)(50) \\ & \mathrm{SA}=225 \pi+1500 \pi \\ & \mathrm{SA}=1725 \pi \approx 5419 \text { sq. ft. } \end{aligned}$ |
| 2. You are asked to paint the ceiling and walls of a room. The room is $10^{\prime}$ wide $\mathrm{x} 15^{\prime}$ long x $10^{\prime}$ high. Calculate the surface area so that you can determine the gallons of paint needed. A gallon of paint covers 175 sq. ft. | $\begin{aligned} & \text { Rectangular prism SA }=2 l \mathrm{w}+2 \mathrm{wh}+2 \mathrm{hl} \\ & \mathrm{SA}=2(15)(10)+2(10)(10)+2(10)(15) \\ & \mathrm{SA}=300+200+300 \\ & \mathrm{SA}=800 \mathrm{ft}^{2} \end{aligned}$ <br> As you are not painting the floor, subtract the SA of the floor. <br> SA (floor) $=$ length $\times$ width $=10 \times 15=150 \mathrm{ft}^{2}{ }^{2}$ <br> $\mathrm{SA}($ painted surfaces $)=800-150=650 \mathrm{ft}^{2}{ }^{2}$ <br> $650 / 175=3.71$ gallons needed. You should purchase 4 gallons. |
| 3. You need to paint two decorative spheres with $d=10^{\prime}$ at the entrance to the new mall. A gallon of paint covers 175 sq. ft. How many gallons of paint will you need? | $\begin{aligned} & \text { One Sphere } \mathrm{SA}=4 \pi \mathrm{r}^{2} \quad \text { Radius }=10 / 2=5 \\ & \mathrm{SA}=4 \pi(5)^{2} \\ & \mathrm{SA}=100 \pi \\ & \mathrm{SA}=314 \mathrm{sq} . \mathrm{ft} . \\ & 2 \text { Spheres }=314+314=628 \text { sq. } \mathrm{ft} . \\ & 628 \div 175=3.59 \text { gallons needed. You should purchase } 4 \text { gallons. } \end{aligned}$ |
| Problems Related, Gen | c Math Concepts Solutions |
| 4. You need fabric to cover a 4-sided pyramid with base sides of $12^{\prime} \&$ slant length of $20^{\prime}$. How many square feet of fabric will you need to cover the pyramid? How many square yards? Note: $1 \mathrm{yd}^{2}=9 \mathrm{ft}^{2}$. | $\begin{aligned} & \text { Pyramid: } \mathrm{SA}=(\text { base area })+1 / 2 l \text { (number of base sides)(b) } \\ & \mathrm{SA}=144+1 / 2(20)(4)(12) \\ & \mathrm{SA}=144+480 \\ & \mathrm{SA}=624 \mathrm{ft.}^{2} \\ & \mathrm{SA}=624 \mathrm{ft.}^{2} \div 9=69.3 \mathrm{yd.}^{2} \end{aligned}$ |
| 5. One soup can has a radius $=3 "$ and height $=4 "$; another soup can has a radius $=4 "$ and height $=3 "$. Which can has a greater total surface area? | $\begin{array}{ll} \hline \text { Can 1: } & \text { Can 2: (Greater surface Area) } \\ \mathrm{SA}=2 \pi\left(3^{2}\right)+2 \pi(3)(4) & \mathrm{SA}=2 \pi\left(4^{2}\right)+2 \pi(4)(3) \\ \mathrm{SA} \approx 57+75 & \mathrm{SA} \approx 101+75 \\ \mathrm{SA} \approx 132 \mathrm{in}^{2} & \mathrm{SA} \approx 176 \mathrm{in} .^{2} \end{array}$ |
| 6. A size 7 regulation basketball has a $d=9.39^{\prime \prime}$. A size 6 regulation basketball has a $\mathrm{d}=9.07$ ". What is the surface area of each basketball? | Ball 1: $\mathrm{r}=4.695$ Ball 2: $\mathrm{r}=4.535$ <br> $\mathrm{SA}=4 \pi\left(4.695^{2}\right)$ $\mathrm{SA}=4 \pi\left(4.535^{2}\right)$ <br> $\mathrm{SA}=4 \pi(22.04)$ $\mathrm{SA}=4 \pi(20.57)$ <br> $\mathrm{SA} \approx 277 \mathrm{in}^{2}$ $\mathrm{SA} \approx 259 \mathrm{in.}^{2}$ |
| Problems PA Core | Math Look Solutions |
| 7. Find the surface area of a cylinder with a diameter of 13.75 ' and a height of 28.45'. | $\begin{aligned} & \text { Cylinder } \mathrm{SA}=2 \pi \mathrm{r}^{2}+2 \pi \mathrm{rh} \\ & \text { radius }=1 / 2 \mathrm{~d}=6.875 \\ & \mathrm{SA}=2 \pi(6.875)^{2}+2 \pi(6.875)(28.45) \\ & \mathrm{SA}=94.53125 \pi+391.1875 \pi \\ & \mathrm{SA}=485.71875 \pi \\ & \mathrm{SA}=1525.9 \mathrm{ft.}^{2} \end{aligned}$ |
| 8. Find the surface area of a sphere that has a diameter of 27.75". | $\begin{aligned} & \text { One Sphere } \mathrm{SA}=4 \pi \mathrm{r}^{2} \\ & \text { Radius }=\mathrm{r}=27.75 / 2=13.875 " \\ & \mathrm{SA}=4 \pi(13.875)^{2} \\ & \mathrm{SA}=770.0625 \pi \\ & \mathrm{SA} \approx 2,419.2 \mathrm{in} .{ }^{2} \end{aligned}$ |
| 9. Find the total surface area of a cone with base diameter of 15.50 " and a height of 22 ". | Cone: $\begin{aligned} & \mathrm{SA}=\pi \mathrm{r}^{2}+\pi \mathrm{r} \quad\left(\sqrt{\left.r^{2}+\mathrm{h}^{2}\right)}\right. \\ & \mathrm{SA}=\pi(7.75)^{2}+\pi(7.75) \sqrt{\left((7.75)^{2}+22^{2}\right)} \\ & \mathrm{SA}=60.0625 \pi+\pi(7.75) \sqrt{60.0625+484} \\ & \mathrm{SA}=60.0625 \pi+\pi(7.75) \sqrt{544.0625} \\ & \mathrm{SA}=60.0625 \pi+\pi(7.75)(23.325) \\ & \mathrm{SA}=60.0625 \pi+\pi(180.769) \\ & \mathrm{SA}=240.83 \pi \\ & \mathrm{SA} \approx 756.2 \mathrm{in.}{ }^{2} \end{aligned}$ |

